2-Microlocal Boundary Value Problems and Their Applications(Functional-Analytic Study of Generalized Functions)

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1. Let M be a real analytic manifold, X a complex neighbourhood of M, and Y a complex hypersurface of X. Let Λ be a regular involutive conic submanifold of $T^*_M X$, Λ^C (resp. $\tilde{\Lambda}$) a complexification (resp. a partial complexification) of Λ in T^*X. Let $\mathcal{M}$ be a coherent $\mathcal{E}_X$-Module defined in a neighbourhood of Λ^C and assume that $Y \longrightarrow X$ is non microcharacteristic along Λ^C for $\mathcal{M}$ (cf. Def. 2.2 of [1] and Def. 2.10.3 of [5]). Set $\Sigma = \Lambda \cap \pi^{-1}(Y)$, and denote by Λ_+ a domain of Λ with boundary Σ. Then we can define the microlocal boundary values along Λ to Σ for $\mathcal{B}^2_\Lambda$-solutions to $\mathcal{M}$. To be precise, there exists the boundary value map

$$bv : R\mathcal{H}om_{\mathcal{E}_X}(\mathcal{M}, \Gamma_{\Lambda_+}(\mathcal{B}^2_\Lambda))|_\Sigma \longrightarrow R\mathcal{H}om_{\mathcal{E}_Y}(\mathcal{M}_Y, \mathcal{B}^2_\Sigma),$$

where $\mathcal{B}^2_\Lambda$ denotes the sheaf of 2-hyperfunctions on Λ due to Kashiwara (cf. [4], [10]), and $\mathcal{M}_Y$ denotes the tangential system on Y of $\mathcal{M}$.

We set

$$\mathcal{C}^2_{\Lambda_+|\tilde{\Lambda}} = \mu hom(\underline{\mathbb{C}}_{\Lambda_+}, \mathcal{C}^h_{\tilde{\Lambda}}) \otimes or_{\Lambda|\tilde{\Lambda}}[\operatorname{codim}\Lambda],$$

where $\mathcal{C}^h_{\tilde{\Lambda}}$ denotes the sheaf of microfunctions with holomorphic parameters on $\tilde{\Lambda}$, and $or_{\Lambda|\tilde{\Lambda}}$ denotes the relative orientation sheaf on Λ. Refer to Kashiwara-Schapira [3] for the bifuncter μhom. This complex of $\mathcal{E}_X$-Modules is a 2-microlocal version of $\mathcal{C}_{\Omega|X}$, the "sheaf" of microfunctions for boundary value problems, which is introduced in Schapira [8]. This allows us to define the "boundary 2-analytic wavefront set" $SS^2_{\Lambda_+}$ for sections of $\Gamma_{\Lambda_+}(\mathcal{B}^2_\Lambda)$, and we can analyse the boundary value map "bv" 2-microlocally in terms of $SS^2_{\Lambda_+}$. In particular, we can show <u>the reflection of 2-microlocal singularity at the boundary</u>. Refer to [13] for the details.

2. Let Ω be an open subset of M with real analytric boundary $N = \{\varphi=0\}$, Y the complexification of N in X. We denote by ρ the natural projection $Y \underset{X}{\times} T^*X \longrightarrow T^*Y$. Take a point $y^* \in T^*_N Y$ and a point $x^* \in T^*_M X \underset{M}{\times} N$ with $\rho(x^*) = y^*$. Let P be a microdifferential operator defined in a neighbourhood of x^* with involutive double characteristics. Precisely we assume that the principal symbol $\sigma(P)$ of P is decomposed by homogeneous holomorphic functions p_1 , p_2 , q :

$$\sigma(P) = q \cdot p_1^{m_1} \cdot p_2^{m_2}$$

in a neighbourhood of x^* and they satisfy the following conditions :

(1) p_1 and p_2 are real valued on $T^*_M X$,

(2) $p_1(x^*) = p_2(x^*) = 0$, $q(x^*) \neq 0$,

(3) $dp_1 \wedge dp_2 \wedge \omega \neq 0$,

(4) $\{p_1, p_2\} = 0$ on $\Lambda = \{p_1 = p_2 = 0\}$,

(5) $\{\varphi, p_i\} \neq 0 \quad (i = 1, 2)$.

In this situation we consider the microlocal boundary value problem

$$(\mathcal{D}) \quad \begin{cases} Pu = 0 \quad \text{at } x^* , \\ (\varphi_X \cdot D_X)^i u|_{\varphi \to +0} = 0 \quad \text{at } y^* \quad (0 \leqq i < \max\{m_1, m_2\}) . \end{cases}$$

Remark that we take here the boundary value $(\varphi_X \cdot D_X)^i u|_{\varphi \to +0}$ of u in the microlocal sense from a neighbourhood of x^* .

Then we have the following results.

We denote by $SS_\Omega(u)$ the boundary analytic wavefront set of u (cf. [8] for the definition of the boundary analytic wavefront set).

<u>Theorem 1.</u>---Let Γ be a real bicharacteristic leaf of Λ passing through x^* . For any solution u of the microlocal boundary value problem $(\mathcal{D})$, there exists a subset $\{x^*_s\}$ of $\Gamma \cap \Sigma$ such that

$SS_\Omega(u) \cap \Gamma$ = the closure in Γ of the union of $\{ b^i_s ; s, i = 1,2 \}$ and some of connected components of $\Gamma_+ \setminus \bigcup \{ b^i_s ; s, i = 1,2 \}$,

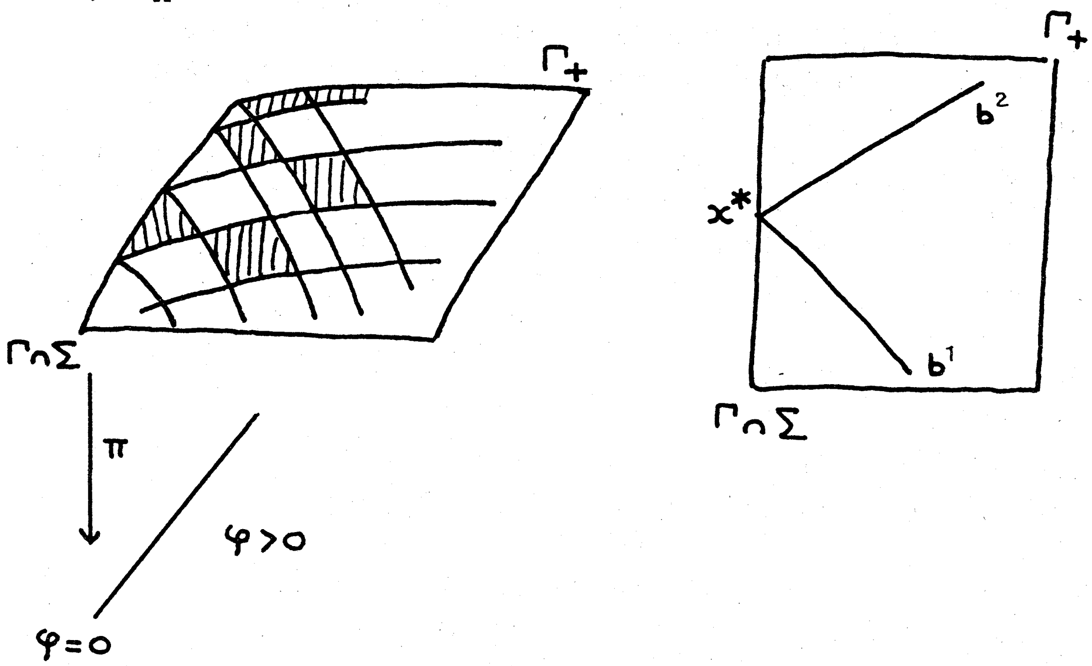
where $\Gamma_+ = \Gamma \underset{M}{\times} \Omega$, and b^i_s denotes the half integral curve of H_{p_i} , the Hamilton vector field of p_i, issued from x^*_s into Γ_+ .

<u>Corollary 2.</u>---For any solution u of the microlocal boundary value problem $(\mathcal{D})$,

$b^1(x^*) \cup b^2(x^*) \not\subset SS(u|_\Omega)$

$\Longrightarrow$

$x^* \notin SS_\Omega(u)$.



Theorem 1 is obtained as an application of the theory of Section 1 (cf. [13]).

<u>Remark.</u>---As for the results in the interior domain for the same operator we refer to Tose [10, 11, 12]. We also refer to Lascar [6] for the similar result as Corollary 2 in the C^∞ category. Note that we assume in Corollary 2 that at least one of the integral curves $b^1(x^*)$, $b^2(x^*)$ is not contained in $SS(u|_\Omega)$ in a neighbourhood of x^*.

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